View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by CERN Document Server

hep-th/0105186

Warped Geometry in Higher Dimensions with an Orbifold Extra Dimension

Masato ITO [1]

Department of Physics, Nagoya University, Nagoya, JAPAN 464-8602

Abstract

We solve the Einstein equations in higher dimensions with warped geometry where an extra dimension is assumed to have orbifold symmetry, S^1/Z_2. The setup considered here is an extension of the 5-dimensional Randall-Sundrum model to $(5+D)$-dimensions, and hidden and observable branes are fixed on the orbifold. It is assumed that the brane tension (self-energy) of each brane with $(4+D)$-dimensional spacetime is anisotropic and that the warped metric function of the 4-dimensions is generally different from the one of the extra D-dimensions. We point out that the form of the warped metric functions and the relations between tensions of two branes depend on the integration constant appearing in the Einstein equations as well as the sign of the bulk cosmological constant.

[1]E-mail address: mito@eken.phys.nagoya-u.ac.jp

1 Introduction

Motivated by the Hořava-Witten model in 11-dimensional theory (M-theory) compactified on the orbifold S^1/Z_2, many model have been proposed using the notion that there are two branes which represent the boundaries of higher dimensional spacetime [1]. Consequently, there have been growing interests among particle physicists and cosmologists toward models with extra dimensions. The recent developments are based on the idea that ordinary matter fields could be confined to a 3-brane world embedded in the higher dimensional space.

Adopting this idea further, there are several proposals which try to relate the Planck scale of observable world to the higher dimensional Planck scale. In the model proposed by N. Arkani-Hamed, S. Dimopoulos and G. Dvali [2], the fundamental scale M_* can be related to the usual 4-dimensional Planck scale M_p via a volume factor, $M_p^2 = M_*^{n+2}R^n$, where R^n is the volume of the compact space and n is the number of extra dimensions. If R is sufficiently large, M_* can be as low as $1TeV$ scale, thus the model gives a possible solution to the gauge hierarchy problem. Furthermore, Randall and Sundrum [4, 5] have presented a static solution to the classical five dimensional Einstein equations with negative bulk cosmological constant (AdS space). The warped metric (factor) in the model is an exponential scaling of the metric along the fifth dimension compactified on the S^1/Z_2 orbifold. This solution appeals to the possibility of an extra dimension limited by two 3-branes with opposite tensions and provides an alternative explanation to the hierarchy problem due to warped factor if the observable brane have the negative brane tension. Both approaches assume that the standard model particles are confined to a 3-brane embedding in higher dimensional spacetime and that gravity exists in the bulk.

An important question concerning these kinds of model is as to whether or not standard 4-dimensional gravity is reproduced on the brane. In the Randall-Sundrum model, even if the fifth dimension is uncompactified, usual gravity is shown to be recovered because of the existence of massless graviton trapped in the brane [5]. Further, another problem is that the stabilization mechanism for size of the extra dimensions is yet unknown. Introducing bulk scalar field which interacts with the branes, several mechanisms have been proposed [10]. On the other hand, the existence of the extra dimensions allows a lot of phenomenology including the production of Kaluza-Klein excitations of graviton at future colliders or their detection in high precision measurements at low energies.

The Randall-Sundrum static solution has been extended to time dependent solutions and their cosmological properties have been extensively studied [7]. In the framework of brane world cosmology, the serious problem emphasized recently is an unusual form of the Friedmann equations in the case of one extra dimension which leads to a particular behavior of the Hubble parameter on the brane. In particular, the Hubble parameter H is proportional to the energy density on the brane instead of the familiar dependence $H \sim \sqrt{\rho}$.

The purpose of this paper is to extend the 5-dimensional Randall-Sundrum model to more higher dimensional case. Since the original Randall-Sundrum model is inspired by superstring theory or M-theory, the version of higher dimensions should be naturally motivated. In this case, we are interested in whether the brane tension of the higher dimensional brane is anisotropic or not and in the relation between the brane tension and the bulk cosmological constant. We study the metric of the $(5 + D)$-dimensional Randall-Sundrum model, where the $(5 + D)$-dimensions are composed of the $(4 + D)$-dimensional spacetime and a dimension compactified on the orbifold S^1/Z_2, $|y| \leq L$. The $(4+D)$-dimensional world resides in the $(3+D)$-brane, and two branes are fixed at $y=0$ and $y=L$. The observable brane we live in is assumed to be the brane at $y=L$ and the hidden brane at $y=0$. The ways of how the metric ansatz is taken are several varieties. In this paper, we consider the case that the 4-dimensional warped metric function $a(y)$ is generally different from the extra D-dimensional warped metric function $c(y)$. Recently, the scenario in which $a(y)$ is equal to $c(y)$ have been discussed [11]. Furthermore, we assume that the brane tension of the $(4+D)$-dimensions is anisotropic, namely, the brane tension of the 4-dimensional spacetime is generally different form the one of the extra D-dimensional space. Based on the above assumptions, we solve the $(5+D)$-dimensional Einstein equation with the bulk cosmological constant and study the form of $a(y)$ and $c(y)$ explicitly. Moreover, we derive the relations between the brane tension of the 4-dimensions and the one of the extra D-dimensions and represent the behavior of each brane tension for the distance between two branes.

This paper is organized as follows. In section 2, the setup considered here is described and generalized Einstein equations with time-dependence are explicitly expressed. In the simplest case of isolated two-brane system, we give the higher dimensional Friedmann-type equation on the brane. In section 3, we solve the static $(5+D)$-dimensional Randall-Sundrum model with the bulk cosmological constant Λ. For each case of $\Lambda<0$, $\Lambda>0$ and $\Lambda=0$, the $(4+D)$-dimensional metric functions can be obtained. We show that the form of warped metric functions and the relation between the brane tensions on the orbifold depend on the integration constant appearing in the Einstein equations as well as the sign of the bulk cosmological constant. A summary and discussion are given in the final section. In an appendix, we review the Kasner solution of the $(4+D)$-dimensional anisotropic cosmological model.

2 The Setup

We consider the higher dimensional spacetime with an orbifold extra dimension. This setup is an extended model of the Randall-Sundrum with 5-dimensional warped metric. The two $(3+D)$-branes with the $(4+D)$-dimensional spacetime embedding in the $(5+D)$-dimensional spacetime are located at $y=0$ and at $y=L$, where y-direction is compactified on the orbifold S^1/Z_2. This $(5+D)$-dimensional model is described by the action

$$S = \int_{-L}^{L} dy \int d^{4+D}x \sqrt{|g|} \left(\frac{1}{2\kappa^2}\mathcal{R} - \Lambda \right) \tag{1}$$

in bulk, where $1/\kappa^2$ is the fundamental gravitational scale and Λ is the bulk cosmological constant.

To solve the Einstein equations, the metric ansatz can be written in the following form

$$\begin{aligned} ds^2 &= n^2(t,y)dt^2 - a^2(t,y)d\vec{x}^2 - b^2(t,y)dy^2 - c^2(t,y)\left(dz_1^2 + \cdots + dz_D^2\right) \\ &\equiv g_{AB}dx^A dx^B\,, \end{aligned} \tag{2}$$

where $A, B = 0, \cdots, 4+D$. We shall use the notation $\{x^\mu\}$ with $\mu = 0, \cdots, 3$ for the coordinates on the 4-dimensional spacetime $\{t, \vec{x}\}$, $x^4 = y$ for an coordinate on the orbifold compactification, and $\{x^a\}$ with $a = 5, \cdots, D+4$ for ones on the extra D-dimensional space

$\{z_1, \cdots, z_D\}$. It is assumed that the distribution of the brane tension and the matter on the brane with $(4+D)$-dimensional spacetime is anisotropic. The Einstein tensor corresponds to

$$G_{AB} = \mathcal{R}_{AB} - \frac{1}{2} g_{AB} \mathcal{R} \,, \tag{3}$$

where $\mathcal{R}_{AB}$ and $\mathcal{R}$ represent the Ricci tensor and the scalar curvature, respectively. The Einstein equation is given by $G_{AB} = \kappa^2 T_{AB}$, where T_{AB} is the energy-momentum tensor. It is assumed that there are contributions to T_{AB} from the bulk and the branes as

$$T_{AB} \;=\; T^{\rm bulk}_{AB} + T^{\rm brane}_{AB} \,. \tag{4}$$

From the bulk we have

$$T^{\rm bulk}_{AB} \;=\; g_{AB}\Lambda \,, \tag{5}$$

where Λ is cosmological constant in the bulk and from two branes

$$\begin{aligned} T^{A,{\rm brane}}_{B} \;=\;& \frac{\delta(y)}{b} diag(\; V_1+\rho_1, V_1-p_1, V_1-p_1, V_1-p_1, 0, \underbrace{V_1^*-p_1^*, \cdots, V_1^*-p_1^*}_{D}\;) \\ &+ \frac{\delta(y-L)}{b} diag(\; V_2+\rho_2, V_2-p_2, V_2-p_2, V_2-p_2, 0, \\ &\qquad\qquad \underbrace{V_2^*-p_2^*, \cdots, V_2^*-p_2^*}_{D}\;) \,. \end{aligned} \tag{6}$$

Here index 1 and 2 denote the brane at $y=0$ and at $y=L$, respectively. V, ρ and p represent the brane tension, the density and the pressure of the matter on each brane, respectively. The superscript $*$ corresponds to quantities in the extra D-dimensional space. Using the metric ansatz Eq.(2), we can write the Einstein equation for each component. The $(0,0)$ component for t-direction is given by

$$\begin{aligned} &\frac{1}{n^2}\left[3\left(\frac{\dot a}{a}\right)^2 + \frac{1}{2}D(D-1)\left(\frac{\dot c}{c}\right)^2 + 3\frac{\dot a}{a}\frac{\dot b}{b} + D\frac{\dot b}{b}\frac{\dot c}{c} + 3D\frac{\dot a}{a}\frac{\dot c}{c}\right] \\ &-\frac{1}{b^2}\left[3\left(\frac{a'}{a}\right)^2 + \frac{1}{2}D(D-1)\left(\frac{c'}{c}\right)^2 + 3\frac{a''}{a} - 3\frac{a'}{a}\frac{b'}{b} + D\frac{c''}{c} + 3D\frac{a'}{a}\frac{c'}{c} - D\frac{b'}{b}\frac{c'}{c}\right] \\ &= \kappa^2\Lambda + \kappa^2\frac{V_1+\rho_1}{b}\delta(y) + \kappa^2\frac{V_2+\rho_2}{b}\delta(y-L) \,. \end{aligned} \tag{7}$$

The (i,i) component for the 3-dimensional space $(i=1,2,3)$ has

$$\begin{aligned} \frac{1}{n^2}\Bigg[&-2\frac{\ddot a}{a} - \frac{\ddot b}{b} - D\frac{\ddot c}{c} - \left(\frac{\dot a}{a}\right)^2 - \frac{1}{2}D(D-1)\left(\frac{\dot c}{c}\right)^2 \\ &\qquad -2\frac{\dot a}{a}\frac{\dot b}{b} - 2D\frac{\dot a}{a}\frac{\dot c}{c} - D\frac{\dot b}{b}\frac{\dot c}{c} + 2\frac{\dot a}{a}\frac{\dot n}{n} + \frac{\dot b}{b}\frac{\dot n}{n} + D\frac{\dot c}{c}\frac{\dot n}{n}\Bigg] \end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{b^2}\left[2\frac{a''}{a}+\frac{n''}{n}+D\frac{c''}{c}+\left(\frac{a'}{a}\right)^2+\frac{1}{2}D(D-1)\left(\frac{c'}{c}\right)^2\right.\\
&\qquad\left.-2\frac{a'}{a}\frac{b'}{b}+2\frac{a'}{a}\frac{n'}{n}-\frac{b'}{b}\frac{n'}{n}+2D\frac{a'}{a}\frac{c'}{c}-D\frac{b'}{b}\frac{c'}{c}+D\frac{c'}{c}\frac{n'}{n}\right]\\
&=-\kappa^2\Lambda-\kappa^2\frac{V_1-p_1}{b}\delta(y)-\kappa^2\frac{V_2-p_2}{b}\delta(y-L)\,.
\end{aligned}
\tag{8}
$$

For the $(4,4)$ component for y-direction compactified on S^1/Z_2 we get

$$
\begin{aligned}
&\frac{1}{n^2}\left[-3\frac{\ddot{a}}{a}-D\frac{\ddot{c}}{c}-3\left(\frac{\dot{a}}{a}\right)^2-\frac{1}{2}D(D-1)\left(\frac{\dot{c}}{c}\right)^2+3\frac{\dot{a}}{a}\frac{\dot{n}}{n}+D\frac{\dot{c}}{c}\frac{\dot{n}}{n}-3D\frac{\dot{a}}{a}\frac{\dot{c}}{c}\right]\\
&+\frac{1}{b^2}\left[3\left(\frac{a'}{a}\right)^2+\frac{1}{2}D(D-1)\left(\frac{c'}{c}\right)^2+3\frac{a'}{a}\frac{n'}{n}+3D\frac{a'}{a}\frac{c'}{c}+D\frac{n'}{n}\frac{c'}{c}\right]=-\kappa^2\Lambda\,.
\end{aligned}
\tag{9}
$$

The (a,a) component for the D-dimensional space $(a=5,\cdots,D+4)$ takes the form

$$
\begin{aligned}
&\frac{1}{n^2}\left[-3\frac{\ddot{a}}{a}-\frac{\ddot{b}}{b}-(D-1)\frac{\ddot{c}}{c}-3\left(\frac{\dot{a}}{a}\right)^2-\frac{1}{2}(D-1)(D-2)\left(\frac{\dot{c}}{c}\right)^2\right.\\
&\qquad\left.-3\frac{\dot{a}}{a}\frac{\dot{b}}{b}-(D-1)\frac{\dot{b}}{b}\frac{\dot{c}}{c}-3(D-1)\frac{\dot{a}}{a}\frac{\dot{c}}{c}+3\frac{\dot{a}}{a}\frac{\dot{n}}{n}+\frac{\dot{b}}{b}\frac{\dot{n}}{n}+(D-1)\frac{\dot{c}}{c}\frac{\dot{n}}{n}\right]\\
&+\frac{1}{b^2}\left[3\frac{a''}{a}+\frac{n''}{n}+(D-1)\frac{c''}{c}+3\left(\frac{a'}{a}\right)^2+\frac{1}{2}(D-1)(D-2)\left(\frac{c'}{c}\right)^2\right.\\
&\qquad\left.-3\frac{a'}{a}\frac{b'}{b}+3\frac{a'}{a}\frac{n'}{n}-\frac{b'}{b}\frac{n'}{n}+3(D-1)\frac{a'}{a}\frac{c'}{c}-(D-1)\frac{b'}{b}\frac{c'}{c}+(D-1)\frac{n'}{n}\frac{c'}{c}\right]\\
&=-\kappa^2\Lambda-\kappa^2\frac{V_1^*-p_1^*}{b}\delta(y)-\kappa^2\frac{V_2^*-p_2^*}{b}\delta(y-L)
\end{aligned}
\tag{10}
$$

and non-diagonal $(0,4)$ component for t- and y-direction is written as

$$
-3\frac{\dot{a}'}{a}+3\frac{n'}{n}\frac{\dot{a}}{a}+3\frac{a'}{a}\frac{\dot{b}}{b}+D\frac{n'}{n}\frac{\dot{c}}{c}-D\frac{\dot{c}'}{c}+D\frac{\dot{b}}{b}\frac{c'}{c}=0\,.
\tag{11}
$$

Here primes (dots) denote the derivative with respect to y (t). Although the functions a,n and c are continuous at the brane, their derivatives with respect to y are discontinuous due to the presence of brane. By matching the coefficients of the delta functions, the $(0,0)$ and (i,i) and (a,a) components of the Einstein equations are subject to the jump conditions on the first derivative of functions. In order to derive jump conditions on a,n and c, we define the function [7]

$$
[f]_x = f(x+0)-f(x-0)
\tag{12}
$$

for arbitrary function f. From Eqs.(7), (8) and (10), the integration over $y\in(-0,+0)$ yields

$$
-3\frac{[a']_0}{a_0}-D\frac{[c']_0}{c_0} = \kappa^2 b_0\left(V_1+\rho_1\right)\,,
$$

$$
\begin{aligned}
2\frac{[a']_0}{a_0}+\frac{[n']_0}{n_0}+D\frac{[c']_0}{c_0} &= -\kappa^2 b_0\left(V_1-p_1\right) , \\
3\frac{[a']_0}{a_0}+\frac{[n']_0}{n_0}+(D-1)\frac{[c']_0}{c_0} &= -\kappa^2 b_0\left(V_1^*-p_1^*\right) .
\end{aligned} \tag{13}
$$

Here we use the notations $n_0 = n(t,0)$, $a_0 = a(t,0)$, $b_0 = b(t,0)$ and $c_0 = c(t,0)$. The jump conditions on n, a and c are rewritten as

$$
\begin{aligned}
\frac{[a']_0}{a_0} &= -\frac{\kappa^2 b_0}{D+3}\left(V_1+\rho_1-D[V_1-V_1^*-p_1+p_1^*]\right) , \\
\frac{[n']_0}{n_0} &= -\frac{\kappa^2 b_0}{D+3}\left(V_1-2\rho_1-3p_1-D[V_1-V_1^*+\rho_1+p_1^*]\right) , \\
\frac{[c']_0}{c_0} &= -\frac{\kappa^2 b_0}{D+3}\left(4V_1-3V_1^*+\rho_1-3p_1+3p_1^*\right) .
\end{aligned} \tag{14}
$$

It is noted that the above jump conditions at $y=0$ depend on the tension, the density and the pressure on the brane as well as the number of the extra dimensions. Similarly, the jump conditions at $y = L$ can be derived. As mentioned later, these jump conditions are used to derive the relations between the brane tensions.

In the Randall-Sundrum model, it is important to study the equation for the cosmological expansion on the brane [7]. Below, we consider the simplest case where two branes are completely isolated each other and give the equations of the higher dimensional cosmological evolution on the brane. This situation corresponds to the limit of $L \to \infty$. Using the jump conditions in Eq.(14), the difference between $y = +0$ and $y = -0$ in Eq.(11) leads to the energy conservation

$$
\begin{aligned}
&\dot{\rho}_1+\frac{9}{D+3}\left(\rho_1+p_1+\frac{D}{9}\left[4V_1-3V_1^*+4\rho_1+3p_1^*\right]\right)\frac{\dot{a}_0}{a_0} \\
&-D\left\{V_1-2\rho_1-3p_1-D\left[V_1-V_1^*+\rho_1+p_1^*\right]\right\}\frac{\dot{c}_0}{c_0}=0
\end{aligned} \tag{15}
$$

on the brane at $y = 0$. In the case $D = 0$, the above equation is reduced to the energy conservation of the 4-dimensional standard cosmology. We now define the average function [7]

$$
\{f\}_x = \frac{f(x+0)+f(x-0)}{2} . \tag{16}
$$

Taking the average between $y = +0$ and $y = -0$ with respect to $(4,4)$ component, we can obtain the Friedmann-type equation on the brane at $y = 0$

$$
\begin{aligned}
&\frac{1}{n_0^2}\left[\frac{\ddot{a}_0}{a_0}+\left(\frac{\dot{a}_0}{a_0}\right)^2+\frac{D}{3}\frac{\ddot{c}_0}{c_0}+\frac{1}{6}D(D-1)\left(\frac{\dot{c}_0}{c_0}\right)^2-\frac{\dot{n}_0}{n_0}\frac{\dot{a}_0}{a_0}-\frac{D}{3}\frac{\dot{n}_0}{n_0}\frac{\dot{c}_0}{c_0}+D\frac{\dot{a}_0}{a_0}\frac{\dot{c}_0}{c_0}\right] \\
&=\frac{1}{3}\kappa^2\Lambda \\
&+\frac{\kappa^4}{4(D+3)^2}\left[V_1+\rho_1-D(V_1-V_1^*-p_1+p_1^*)\right] \\
&\qquad\qquad\times\left[2V_1-\rho_1-3p_1+D(2V_1-V_1^*-2p_1+p_1^*)\right]
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{D\kappa^4}{24(D+3)^2}\left[4V_1-3V_1^*+\rho_1-3p_1+3p_1^*\right]\\
&\qquad\qquad\times\left[-2V_1+3V_1^*-5\rho_1-3p_1-3p_1^*+D(2V_1-V_1^*-\rho_1-3p_1+p_1^*)\right]\\
&+\left(1-3\frac{V_1-p_1}{V_1+\rho_1}\right)\left(\frac{\{a'\}_0}{a_0b_0}\right)^2+\frac{D}{6}\left(D-1-2D\frac{V_1^*-p_1}{V_1^*+\rho_1}\right)\left(\frac{\{c'\}_0}{c_0b_0}\right)^2\\
&+D\left(1-\frac{V_1+V_1^*-p_1-p_1^*}{V_1+\rho_1}\right)\frac{\{c'\}_0\{a'\}_0}{a_0c_0b_0^2}
\end{aligned}
\tag{17}
$$

on the brane at $y=0$. Imposing the orbifold symmetry $y\sim -y$, we have $\{f'\}=0$. Then we can drop all terms involving the average in Eq.(17). We have also fixed the time in such a way that $n_0=1$. This corresponds to the usual choice of time in conventional cosmology. We introduce the Hubble parameters $H_a\equiv \dot{a}/a$ and $H_c\equiv \dot{c}/c$ for the two scale factors on the brane at $y=0$. Here it is assumed that after stabilizing the radion b_0 the matter on the brane is isotropic and that radiation is dominated. This leads to $p_1=p_1^*$ and $\rho_1=(D+3)p_1$. Then, the cosmological evolution equation on the brane at $y=0$ becomes

$$
\begin{aligned}
&\dot{H}_a+2H_a^2+\frac{D}{3}\dot{H}_c+\frac{1}{6}D(D+1)H_c^2+DH_aH_c\\
&=\frac{1}{3}\kappa^2\Lambda+\frac{\kappa^4}{4(D+3)^2}\left[V_1+\rho_1-D(V_1-V_1^*)\right]\\
&\qquad\qquad\times\left[2V_1-\rho_1\frac{6+D}{3+D}+D\left(2V_1-V_1^*-\rho_1\frac{1}{3+D}\right)\right]\\
&+\frac{D\kappa^4}{24(D+3)^2}\left[4V_1-3V_1^*+\rho_1\right]\\
&\qquad\qquad\times\left[-2V_1+3V_1^*-\rho_1\frac{21+D}{3+D}+D\left(2V_1-V_1^*-\rho_1\frac{5+D}{3+D}\right)\right].
\end{aligned}
\tag{18}
$$

The energy conservation equation and the cosmological evolution equation on the brane at $y=L$ can be obtained by the same procedures as mentioned above.

Since the cosmology equations obtained here corresponds to the completely isolated brane-system, it implies that the matter on one brane has nothing to do with the matter on another brane. However, in the case of finite L two branes are closely related, so that the matter on one brane is constrained by the matter on another brane. To study the cosmology equations constrained by two branes [8], the relation between functions n, a, b and c must be led by integrating out Eq.(11). As for this point, we are going to provide the analysis in detail of the cosmological evolution in the setup presented in this paper [9].

3 Static solutions

We can obtain the static Randall-Sundrum-type solution by setting the density and the pressure of matter to zero. Note that the functions n, a and c have time-independence and preserving Poincaré invariance in the $(1+3)$-dimensional metric

$$
n(y)=a(y),\qquad b=1\,,
\tag{19}
$$

where b is normalized to be unity since it is assumed that the size in y-direction compactified on the orbifold is stabilized via some mechanisms. We consider that the 4-dimensional warped metric function $a(y)$ is generally different from the extra D-dimensional $c(y)$. Following from Eq.(7) to (10), the Einstein equations in the bulk are given by

$$12\left(\frac{a'}{a}\right)^2 + D(D-1)\left(\frac{c'}{c}\right)^2 + 8D\frac{a'}{a}\frac{c'}{c} = -2\kappa^2\Lambda\,, \tag{20}$$

$$6\left(\frac{a'}{a}\right)^2 + D(D-1)\left(\frac{c'}{c}\right)^2 + 6\frac{a''}{a} + 2D\frac{c''}{c} + 6D\frac{a'}{a}\frac{c'}{c} = -2\kappa^2\Lambda\,, \tag{21}$$

$$12\left(\frac{a'}{a}\right)^2 + (D-1)(D-2)\left(\frac{c'}{c}\right)^2 + 8\frac{a''}{a} + 2(D-1)\frac{c''}{c} + 8(D-1)\frac{a'}{a}\frac{c'}{c} = -2\kappa^2\Lambda\,, \tag{22}$$

where we used the fact that $(0,0)$ component is equal to (a,a) component.

Here we can derive the solution of the 5-dimensional classical Einstein equation to be the Randall-Sundrum model. Setting $D = 0$ and neglecting Eq.(22) coming from the (a,a) component for the extra D-dimensions, we have

$$\left(\frac{a'}{a}\right)^2 = \frac{a''}{a} = -\frac{\kappa^2\Lambda}{6}\,. \tag{23}$$

For $\Lambda < 0$, the S^1/Z_2 orbifold symmetric solution is of the form

$$a(y) = e^{\pm m_0|y|}\,, \tag{24}$$

where

$$m_0 = \sqrt{\frac{-\kappa^2\Lambda}{6}}\,. \tag{25}$$

From Eq.(14), the jump conditions of $a(y)$ at $y = 0$ and $y = L$ lead to

$$V_1 = -V_2 = \mp\sqrt{\frac{-\Lambda}{6\kappa^2}}\,, \tag{26}$$

where the sign of upper and lower corresponds to the sign in Eq.(24), respectively. Thus the brane tensions V_1, V_2 at $y = 0$ and $y = L$ have the opposite sign each other. When V_2 is negative, the warped metric becomes

$$ds^2 = e^{-2m_0|y|}g_{\mu\nu}dx^\mu dx^\nu - dy^2\,, \tag{27}$$

where $g_{\mu\nu} = diag(+,-,-,-)$. By using this warped metric, Randall and Sundrum proposed an alternative solution to the hierarchy problem. This solution appeals to the possibility of an extra dimension limited by two branes with positive and negative tensions. Further, the resolution of the hierarchy problem is possible provided that the observable brane at $y = L$ is the one with the negative tension. This model insists that the hierarchy has its origin of the geometry of the extra dimension.

We are interested in the solutions of Randall-Sundrum model embedding in $(5+D)$-dimensions with an orbifold compactification. The feature of this setup is that the warped metric function $a(y)$ of the 4-dimensional spacetime is generally different from the one $c(y)$ of the extra D-dimensional space. After some algebra, we can rewrite the appropriate linear combination of Eqs.(20), (21) and (22) as

$$12\left(\frac{a'}{a}\right)^2+D(D-1)\left(\frac{c'}{c}\right)^2+8D\frac{a'}{a}\frac{c'}{c} = -2\kappa^2\Lambda\,, \tag{28}$$

$$\frac{a''}{a}+3\left(\frac{a'}{a}\right)^2+D\frac{a'}{a}\frac{c'}{c} = -\frac{2\kappa^2}{D+3}\Lambda\,, \tag{29}$$

$$\frac{c''}{c}+(D-1)\left(\frac{c'}{c}\right)^2+4\frac{a'}{a}\frac{c'}{c} = -\frac{2\kappa^2}{D+3}\Lambda\,. \tag{30}$$

To study the behavior of $a(y)$ and $c(y)$, let us consider three cases of $\Lambda<0$, $\Lambda>0$ and $\Lambda=0$, separately.

3.1 The solutions for $\Lambda < 0$

In this case, in order to obtain the solution of the Einstein equations, we perform the changes of variables which allow the exact solution of the equations in Eqs.(28)-(30), we define $A(y)$ and $C(y)$ by

$$a(y)=e^{A(y)}, \quad c(y)=e^{C(y)}\,. \tag{31}$$

Further, defining the parameter $\omega\equiv 2\kappa^2$ where we use. From Eqs.(28), (29) and (30), we obtain

$$\begin{aligned} 12(A')^2+D(D-1)(C')^2+8DA'C' &= -\omega\Lambda\,,\\ A''+4(A')^2+DA'C' &= -\frac{\omega\Lambda}{D+3}\,,\\ C''+D(C')^2+4A'C' &= -\frac{\omega\Lambda}{D+3}\,. \end{aligned} \tag{32}$$

We introduce the new variable Y

$$dY=\mp e^{-4A-DC}dy\,. \tag{33}$$

This replacement of the variable is similar to the procedure of seeking the Kasner solution with time-dependence in the higher dimensional cosmology in Ref.[3]. Thus we get

$$12\left(\frac{dA}{dY}\right)^2+D(D-1)\left(\frac{dC}{dY}\right)^2+8D\frac{dA}{dY}\frac{dC}{dY} = -\omega\Lambda e^{8A+2DC}\,, \tag{34}$$

$$\frac{d^2A}{dY^2}=\frac{d^2C}{dY^2} = -\frac{\omega\Lambda}{D+3}e^{8A+2DC}\,. \tag{35}$$

Note that the above equations are unchanged when lower or upper sign including in Eq.(33) is taken in either case. Hence, we can immediately write the integral of Eq.(35)

$$A-C=P_1Y+P_2\,, \tag{36}$$

where P_1 and P_2 are the integration constants. To solve the differential equation of Eq.(34), we define a new variable [3]

$$Z = 8A + 2DC\,, \tag{37}$$

Eq.(34) is translated as

$$\frac{d^2Z}{dY^2} \quad = \quad -\frac{2(D+4)\omega\Lambda}{D+3}e^Z\,. \tag{38}$$

Integrating this equation, we obtain

$$\left(\frac{dZ}{dY}\right)^2 = -\frac{4(D+4)\omega\Lambda}{D+3}e^Z + P_3\,, \tag{39}$$

where P_3 is a integration constant. However, P_3 is a function of P_1. This is because substitution of Eq.(37) into Eq.(39) leads to

$$P_3 = \frac{16D}{D+3}\left(\frac{dA}{dY} - \frac{dC}{dY}\right)^2 = \frac{16D}{D+3}P_1^2\,. \tag{40}$$

Therefore, we have

$$\left(\frac{dZ}{dY}\right)^2 = -\frac{4(D+4)\omega\Lambda}{D+3}e^Z + \frac{16D}{D+3}P_1^2\,. \tag{41}$$

Since P_1 and P_2 are determined by the initial condition, the values are expected to be determined via some dynamics of underlying physics. Below, we pointed out that types of the solutions of Eq.(41) are obtained by depending on whether P_1 is nonvanishing or vanishing.

First, we consider the case of $P_1 \neq 0$ for the negative bulk cosmological constant. Eq.(41) can be simply solved as

$$e^Z = \frac{4DP_1^2}{(D+4)\omega|\Lambda|}\frac{1}{\sinh^2\left(2\sqrt{\frac{D}{D+3}}P_1Y\right)}\,. \tag{42}$$

Using Eqs.(36) and (37), we get

$$\begin{aligned} a = e^A \quad &= \quad \left[\frac{4DP_1^2}{(D+4)\omega|\Lambda|}\frac{e^{2D(P_1Y+P_2)}}{\sinh^2\left(2\sqrt{\frac{D}{D+3}}P_1Y\right)}\right]^{\frac{1}{2(4+D)}}\,, \\ c = e^C \quad &= \quad \left[\frac{4DP_1^2}{(D+4)\omega|\Lambda|}\frac{e^{-8(P_1Y+P_2)}}{\sinh^2\left(2\sqrt{\frac{D}{D+3}}P_1Y\right)}\right]^{\frac{1}{2(4+D)}}\,. \end{aligned} \tag{43}$$

To write the above equations in terms of y, we need to change the variable Y into y. Eq.(33) becomes

$$dY = \mp\sqrt{\frac{D+4}{4DP_1^2}\omega|\Lambda|}\,\sinh\left(2\sqrt{\frac{D}{D+3}}\,P_1Y\right)\,dy \tag{44}$$

which leads to the relation between Y and y

$$\exp\left[2\sqrt{\frac{D}{D+3}}P_1 Y\right] = \coth\left(\frac{1}{2}\sqrt{\frac{D+4}{D+3}}\omega|\Lambda|\,(y+y_0)\right)\,, \tag{45}$$

where y_0 is a integration constant and positive sign in Eq.(44) is taken. After substitution of Eq.(45) into Eq.(43), the functions a and c are described in terms of y

$$\begin{aligned} a(y) &= \left(\frac{4DP_1^2}{(D+4)\omega|\Lambda|}\right)^{\frac{1}{2(D+4)}} \\ &\times\left[\coth^{\frac{\sqrt{D(D+3)}}{2}}\left(\frac{1}{2}\sqrt{\frac{D+4}{D+3}}\omega|\Lambda|\,(|y|+y_0)\right)\sinh\left(\sqrt{\frac{D+4}{D+3}}\omega|\Lambda|\,(|y|+y_0)\right)\right]^{\frac{1}{D+4}}\,, \\ c(y) &= \left(\frac{4DP_1^2}{(D+4)\omega|\Lambda|}\right)^{\frac{1}{2(D+4)}} \\ &\times\left[\tanh^{2\sqrt{\frac{D+3}{D}}}\left(\frac{1}{2}\sqrt{\frac{D+4}{D+3}}\omega|\Lambda|\,(|y|+y_0)\right)\sinh\left(\sqrt{\frac{D+4}{D+3}}\omega|\Lambda|\,(|y|+y_0)\right)\right]^{\frac{1}{D+4}}\,, \end{aligned} \tag{46}$$

where a and c respect the Z_2 symmetry, $y \sim -y$. The constant P_2 in Eq.(36) don't appear in the expression of metric functions $a(y)$ and $c(y)$, namely, P_2 could be set to zero since P_2 can be absorbed into a redefinition of the extra D-dimensional coordinates. Moreover, the coefficient including no P_2 in $a(y)$ and $c(y)$ is considered to be physical irrelevant since we are only interested in the behavior with respect to y. From Eq.(14), the jump conditions at $y=0$ lead to

$$\begin{aligned} \frac{2\sqrt{|\lambda|}}{4+D}\frac{2\cosh\sqrt{|\lambda|}y_0-\sqrt{D(D+3)}}{2\sinh\sqrt{|\lambda|}y_0} &= -\frac{\kappa^2}{D+3}\left(V_1 - D[V_1-V_1^*]\right)\,, \\ \frac{2\sqrt{|\lambda|}}{4+D}\frac{\cosh\sqrt{|\lambda|}y_0+2\sqrt{\frac{D+3}{D}}}{\sinh\sqrt{|\lambda|}y_0} &= -\frac{\kappa^2}{D+3}\left(4V_1-3V_1^*\right)\,, \end{aligned} \tag{47}$$

where we define

$$\lambda = \frac{D+4}{D+3}\omega\Lambda\,. \tag{48}$$

Furthermore, the jump conditions at $y=L$ lead to

$$\begin{aligned} \frac{2\sqrt{|\lambda|}}{4+D}\frac{\sinh\sqrt{|\lambda|}y_0\,(2\cosh\sqrt{|\lambda|}y_0-\sqrt{D(D+3)}\cosh\sqrt{|\lambda|}L)}{\cosh 2\sqrt{|\lambda|}y_0-\cosh 2\sqrt{|\lambda|}L} &= \frac{\kappa^2}{D+3}\left(V_2-D[V_2-V_2^*]\right)\,, \\ \frac{2\sqrt{|\lambda|}}{4+D}\frac{2\sinh\sqrt{|\lambda|}y_0\,(\cosh\sqrt{|\lambda|}y_0+2\sqrt{\frac{D+3}{3}}\cosh\sqrt{|\lambda|}L)}{\cosh 2\sqrt{|\lambda|}y_0-\cosh 2\sqrt{|\lambda|}L} &= \frac{\kappa^2}{D+3}(4V_2-3V_2^*)\,. \end{aligned} \tag{49}$$

From Eqs.(47) and (49), the brane tensions at $y = 0$ and $y = L$ are expressed as

$$
\begin{aligned}
V_1 &= -\frac{2\sqrt{|\lambda|}}{\kappa^2(4+D)} \frac{2(D+3)\cosh\sqrt{|\lambda|}y_0 + \sqrt{D(D+3)}}{2\sinh\sqrt{|\lambda|}y_0}\,, \\
V_1^* &= -\frac{2\sqrt{|\lambda|}}{\kappa^2(4+D)} \frac{(D+3)\cosh\sqrt{|\lambda|}y_0 - 2\sqrt{\frac{D+3}{D}}}{\sinh\sqrt{|\lambda|}y_0}\,, \\
V_2 &= \frac{2\sqrt{|\lambda|}}{\kappa^2(4+D)} \frac{\{2(D+3)\cosh\sqrt{|\lambda|}y_0 + \sqrt{D(D+3)}\cosh\sqrt{|\lambda|}L\}\sinh\sqrt{|\lambda|}y_0}{\cosh 2\sqrt{|\lambda|}y_0 - \cosh 2\sqrt{|\lambda|}L}\,, \\
V_2^* &= \frac{2\sqrt{|\lambda|}}{\kappa^2(4+D)} \frac{2\{(D+3)\cosh\sqrt{|\lambda|}y_0 - 2\sqrt{\frac{D+3}{D}}\cosh\sqrt{|\lambda|}L\}\sinh\sqrt{|\lambda|}y_0}{\cosh 2\sqrt{|\lambda|}y_0 - \cosh 2\sqrt{|\lambda|}L}\,.
\end{aligned}
\tag{50}
$$

From the above equations, the integration constant y_0 is expressed as

$$
y_0 = \frac{1}{\sqrt{|\lambda|}} \operatorname{arc\,sinh}\left[\frac{1}{\kappa(V_1^* - V_1)}\sqrt{\frac{2(D+4)}{D}|\Lambda|}\right]. \tag{51}
$$

The sign of y_0 depends on the sign of the difference between V_1 and V_1^*. If $V_1 > V_1^*$, y_0 becomes negative, and Eq.(46) leads to the conclusion that $a(y)$ has singular point as long as $D \neq 1$. To avoid this a singular point over $|y| \leq L$, $-y_0 > L$ is required.

From Eq.(50), the brane tensions V_2, V_2^* of the observable brane at $y = L$ can be described in terms of V_1, V_1^* of the hidden brane at $y = 0$. The ratios V_1/V_1^* and V_2/V_2^* cannot be unity as long as D is positive integer. Thus, each brane tension becomes anisotropic in this setup, and each brane tension is closely related each other due to the presence of two branes. Taking the limit $L \gg y_0$ in infinite orbifold extra dimension where observable brane is fixed far away from origin [6], the ratio V_2 to V_2^* of the observable brane becomes

$$
\frac{V_2}{V_2^*} = -\frac{1}{4}D\,. \tag{52}
$$

V_2 and V_2^* have opposite sign each other and the magnitude of ratio depends on the number of the extra D-dimensions.

Next, let us consider the case $P_1 = 0$. There exists the solution of Eq.(41) with the negative bulk cosmological constant. Integrating it we obtain

$$
e^Z = \frac{D+3}{(D+4)\omega|\Lambda|Y^2}\,. \tag{53}
$$

Using Eqs.(36) and (37), we have

$$
\begin{aligned}
e^A &= \left[\frac{D+3}{(D+4)\omega|\Lambda|}\frac{e^{2DP_2}}{Y^2}\right]^{\frac{1}{2(4+D)}}\,, \\
e^C &= \left[\frac{D+3}{(D+4)\omega|\Lambda|}\frac{e^{-8P_2}}{Y^2}\right]^{\frac{1}{2(4+D)}}\,.
\end{aligned}
\tag{54}
$$

Eq.(33) leads to the relation between Y and y

$$Y = Y_0 \exp\left[\mp\sqrt{\frac{D+4}{D+3}\omega|\Lambda|}\, y\right] , \tag{55}$$

where Y_0 is an integration constant. After substitution of Eq.(55) into Eq.(54), the a and c are described in terms of y

$$a(y) = c(y) = \exp\left[\pm\sqrt{\frac{\omega|\Lambda|}{(D+3)(D+4)}}\,|y|\right] . \tag{56}$$

Hence the sign of upper and lower correspond to the sign in Eq.(55) and both a and c are normalized to be unity at $y = 0$. The jump conditions of $y = 0$ and $y = L$ lead to

$$\begin{aligned} \pm 2\sqrt{\frac{\omega|\Lambda|}{(D+3)(D+4)}} &= -\frac{\kappa^2}{D+3}\left(V_1 - D[V_1 - V_1^*]\right) = -\frac{\kappa^2}{D+3}\left(4V_1 - 3V_1^*\right) \\ &= \frac{\kappa^2}{D+3}\left(V_2 - D[V_2 - V_2^*]\right) = \frac{\kappa^2}{D+3}\left(4V_2 - 3V_2^*\right) , \end{aligned} \tag{57}$$

then, the brane tensions are given by

$$V_1 = V_1^* = -V_2 = -V_2^* = \mp\frac{2}{\kappa}\sqrt{2\frac{D+3}{D+4}|\Lambda|} . \tag{58}$$

In this case, we find that $a(y)$ is equal to $c(y)$ and the brane tension of each brane is automatically guaranteed to be isotopic. The lower sign in Eq.(58) corresponds to the case that the brane tension of the observable brane at $y = L$ is negative. Consequently, the warped metric function become the exponential damping factor since the lower sign (negative) in Eq.(56) is selected. This situation is similar to the five dimensional Randall-Sundrum solution. Likewise Randall-Sundrum scenario, the hierarchy between physical mass scale $m_{\rm hid}$ on the hidden brane at $y = 0$ and $m_{\rm obs}$ on the observable brane at $y = L$ can be generated from the warped metric.

In the case of the negative bulk cosmological constant, whether the integration constant P_1 is nonzero or zero determines the form of the warped metric function. If $P_1 \neq 0$, the warped metric functions $a(y)$ and $c(y)$ have differently the form of hyperbolic function and the brane tension is anisotropic. If $P_1 = 0$ and the observable brane has negative brane tension, both the warped metric functions have the same form of exponential damping factor and the brane tension is isotropic. Thus, whether the brane tension on the orbifold is isotropic or anisotropic depends on the value of P_1. Namely, the integration constant P_1 controls the solution of the Einstein equation in bulk and it is expected that P_1 is determined by initial configuration of brane world in this setup. Hence it is assumed that the dynamical mechanism of fixing the value of P_1 is unknown.

3.2 The solutions for $\Lambda > 0$

When the bulk cosmological constant Λ is positive, Eq.(41) implies that P_1 should be nonzero. Solving this equation, we obtain

$$e^Z = \frac{4DP_1^2}{(D+4)\omega\Lambda}\frac{1}{\cosh^2\left(2\sqrt{\frac{D}{D+3}}P_1Y\right)}\,. \tag{59}$$

Furthermore, we have

$$\begin{aligned} e^A &= \left[\frac{4DP_1^2}{(D+4)\omega\Lambda}\frac{e^{2D(P_1Y+P_2)}}{\cosh^2\left(2\sqrt{\frac{D}{D+3}}P_1Y\right)}\right]^{\frac{1}{2(4+D)}}\,, \\ e^C &= \left[\frac{4DP_1^2}{(D+4)\omega\Lambda}\frac{e^{-8(P_1Y+P_2)}}{\cosh^2\left(2\sqrt{\frac{D}{D+3}}P_1Y\right)}\right]^{\frac{1}{2(4+D)}}\,. \end{aligned} \tag{60}$$

Following Eq.(33),

$$\exp\left[2\sqrt{\frac{D}{D+3}}P_1Y\right] = \mp\tan\left(\frac{1}{2}\sqrt{\frac{D+4}{D+3}\omega\Lambda}\,(y+y_1)\right)\,, \tag{61}$$

where y_1 is a integration constant. By imposing the orbifold symmetry, we can rewrite a and c in terms of y

$$\begin{aligned} a(y) &= \left(\frac{4DP_1^2}{(D+4)\omega\Lambda}\right)^{\frac{1}{2(D+4)}}\left[\tan^{\frac{\sqrt{D(D+3)}}{2}}\left(\frac{\sqrt{\lambda}}{2}\,(|y|+y_1)\right)\,\sin\left(\sqrt{\lambda}\,(|y|+y_1)\right)\right]^{\frac{1}{D+4}}\,, \\ c(y) &= \left(\frac{4DP_1^2}{(D+4)\omega\Lambda}\right)^{\frac{1}{2(D+4)}}\left[\cot^{2\sqrt{\frac{D+3}{D}}}\left(\frac{\sqrt{\lambda}}{2}\,(|y|+y_1)\right)\,\sin\left(\sqrt{\lambda}\,(|y|+y_1)\right)\right]^{\frac{1}{D+4}}\,, \end{aligned} \tag{62}$$

where λ is defined in Eq.(48) and P_2 can be set to zero as mentioned previously. The jump conditions lead to the brane tensions as follows

$$\begin{aligned} V_1 &= -\frac{2\sqrt{\lambda}}{\kappa^2(4+D)}\frac{2(D+3)\cos\sqrt{\lambda}y_1 - \sqrt{D(D+3)}}{2\sin\sqrt{\lambda}y_1}\,, \\ V_1^* &= -\frac{2\sqrt{\lambda}}{\kappa^2(4+D)}\frac{(D+3)\cos\sqrt{\lambda}y_1 + 2\sqrt{\frac{D+3}{D}}}{\sin\sqrt{\lambda}y_1}\,, \\ V_2 &= \frac{2\sqrt{\lambda}}{\kappa^2(4+D)}\frac{\sinh\sqrt{\lambda}y_1\,\left(2(D+3)\cosh\sqrt{\lambda}y_1 + \sqrt{D(D+3)}\cosh\lambda L\right)}{\cosh 2\sqrt{\lambda}y_1 - \cosh 2\sqrt{\lambda}L}\,, \\ V_2^* &= \frac{2\sqrt{\lambda}}{\kappa^2(4+D)}\frac{2\sinh\sqrt{\lambda}y_1\,\left((D+3)\cosh\sqrt{\lambda}y_1 - 2\sqrt{\frac{D+3}{D}}\cosh\sqrt{\lambda}L\right)}{\cosh 2\sqrt{\lambda}y_1 - \cosh 2\sqrt{\lambda}L}\,. \end{aligned} \tag{63}$$

Using the above equations, the integration constant y_1 is expressed as

$$y_1 = \frac{1}{\sqrt{\lambda}} \arcsin\left[\frac{1}{\kappa(V_1 - V_1^*)}\sqrt{\frac{2(D+4)}{D}\Lambda}\right] . \tag{64}$$

The sign of y_1 depends on the sign of the difference between V_1 and V_1^*. If $V_1^* > V_1$, y_1 becomes negative and it is found that $c(y)$ has singularity. Moreover, Eq.(61) leads to the constraint on location of $y = L$

$$L < \pi\sqrt{\frac{D+3}{(D+4)\omega\Lambda}} - y_1 . \tag{65}$$

Likewise the case of negative bulk cosmological constant, the brane tension becomes anisortopic. The warped metric functions obtained here have differently the form of trigonometric functions.

3.3 The solutions for $\Lambda = 0$

In the case of zero bulk cosmological constant, the metric functions are quite similar to the Kasner solution in higher dimensional cosmology in an appendix. We take the power-law form by taking account for the orbifold symmetry

$$\begin{aligned} a(y) &= \left(\frac{|y|}{y_2} + 1\right)^k , \\ c(y) &= \left(\frac{|y|}{y_2} + 1\right)^l , \end{aligned} \tag{66}$$

where a and c are normalized to be unity at $y = 0$ and y_2 is a constant to be determined later. Substituting the above equations into Eq.(20)-(22), we obtain two equations for the exponents

$$\begin{aligned} 4k + Dl &= 1 , \\ 4k^2 + Dl^2 &= 1 . \end{aligned} \tag{67}$$

Solving the above equations, k and l are given by

$$\begin{aligned} k &= \frac{2 \pm \sqrt{D(D+3)}}{2(D+4)} , \\ l &= \frac{D \mp 2\sqrt{D(D+3)}}{D(D+4)} . \end{aligned} \tag{68}$$

Here, we cannot determine whether the sign including in Eq.(68) is lower or upper at this stage.

From Eq.(14), the jump conditions on a and c at $y = 0$ yield

$$\begin{aligned} \frac{2k}{y_2} &= -\frac{\kappa^2}{D+3}\left(V_1 - D[V_1 - V_1^*]\right) , \\ \frac{2l}{y_2} &= -\frac{\kappa^2}{D+3}(4V_1 - 3V_1^*) , \end{aligned} \tag{69}$$

furthermore, the jump conditions at $y = L$ lead to

$$
\begin{aligned}
\frac{2ky_2}{L^2 - y_2^2} &= -\frac{\kappa^2}{D+3}\left(V_2 - D[V_2 - V_2^*]\right) , \\
\frac{2ly_2}{L^2 - y_2^2} &= -\frac{\kappa^2}{D+3}(4V_2 - 3V_2^*) .
\end{aligned} \tag{70}
$$

Using the above equations, the brane tensions are given by

$$
\begin{aligned}
V_1 &= -\frac{2}{\kappa^2 y_2}(1-k) , \\
V_1^* &= -\frac{2}{\kappa^2 y_2}(1-l) , \\
V_2 &= -\frac{2y_2}{\kappa^2(L^2 - y_2^2)}(1-k) , \\
V_2^* &= -\frac{2y_2}{\kappa^2(L^2 - y_2^2)}(1-l) .
\end{aligned} \tag{71}
$$

Following the constraint on exponents in Eq.(67), the constant y_2 becomes

$$
y_2 = -\frac{2(D+3)}{\kappa^2(4V_1 + DV_1^*)} . \tag{72}
$$

Consequently, the ratios V_1/V_1^* and V_2/V_2^* of each brane cannot be unity for arbitrary positive D, namely, the brane tension becomes anisotropic. From Eq.(71), we have $V_1/V_1^* = V_2/V_2^* = (1-k)/(1-l)$, it means that the ratio of each brane is equal. Furthermore, since k and l cannot be beyond unity, the sign of both V_1 and V_1^* is always negative and the relative size between L and y_2 determines the sign of both V_2 and V_2^*. Taking the limit $L \gg y_2$ to be infinitely fixed observable brane [6] , both V_2 and V_2^* approaches zero. Moreover, if the extra D-dimensional space has infinite dimension, taking the limit of $D \to \infty$, $k \to \pm 1/2$ and $l \to 0$, and the ratio becomes

$$
\frac{V_1}{V_1^*} = \frac{V_2}{V_2^*} = \frac{1}{2}, \frac{3}{2} . \tag{73}
$$

Thus, V_1 and V_1^* have same sign and V_2 and V_2^* also do.

For zero bulk cosmological constant, the warped metric functions $a(y)$ and $c(y)$ have differently the form of power-law whose exponents are similar to the constraints appearing the Kasner solution of higher dimensional cosmology. Furthermore, in the case of an infinite orbifold dimension the brane tension of the observable brane becomes zero.

4 Summary and Discussion

We study the warped metric in the $(5+D)$-dimensional Einstein equation with an extra dimension compactified on the orbifold S^1/Z_2, where two $(3+D)$-branes are fixed on y-direction in orbifold compactification, the hidden brane at $y = 0$ and the observable brane at $y = L$. It is assumed that the energy-momentum tensor on the brane have anisotropic brane tension, anisotropic density and anisotropic pressure. With ansatz metric in this paper, the warped metric function $a(y)$ of the 4-dimensions is generally different from the one $c(y)$ of the D-dimensions. We solved the Einstein equations in this setup.

For the negative bulk cosmological constant, whether the integration constant P_1 in differential equation coming from the Einstein equation is non-zero or zero controls the form of $a(y)$ and $c(y)$. If $P_1 \neq 0$, $a(y)$ and $c(y)$ have differently the form of hyperbolic function, we pointed out that the brane tension becomes anisotropic. If $P_1 = 0$, $a(y)$ and $c(y)$ have the same form of exponential factor and the brane tension becomes isotropic. Furthermore, if the observable brane has negative brane tension, the warped metric function is the exponential damping factor and this case is similar to 5-dimensional Randall-Sundrum scenario. For positive bulk cosmological constant, the integration constant P_1 is required to be nonzero in order for the solution of differential equation to exist. The $a(y)$ and $c(y)$ have differently the form of trigonometric function and the brane tension become anisotropic. On the other hand, zero bulk cosmological constant leads that $a(y)$ and $c(y)$ have differently the form of power-law whose exponents are constrained and that the brane tension becomes anisotropic.

As mentioned above, the dynamics of differential equation depend on the integration constant P_1 which is determined by initial condition. The mechanism of fixing the value of P_1 is unknown, however, it is expected that this is determined via dynamics of underlying physics, namely, the initial configuration of brane world. In section 2, we derived the cosmological evolution equation in the isolated two-branes system embedding in $(5 + D)$ dimensions with warped metric, We are going to study the cosmogy constrained by two branes in this setup. Moreover, it is necessary to explore the massless gravitational fluctuations about our classical solution obtained here and to study the stabilization mechanism for compactification. Finally, we expect that this setup may be connected to the D-brane configuration in the framework of superstring.

Acknowledgments

We would like to thank T. Matsuoka (Koggakan Univ.) and M. Matsunaga (Mie Univ.) for the hospitality.

Appendix: Review of the Kasner Solution

In appendix, we review the Kasner solution in higher dimensional cosmology [3, 12]. The original Kasner cosmology is famous for an example of the anisotropic 4-dimensional cosmological model and the metric of the Kasner form is

$$ds^2 = dt^2 - t^{2p}dx_1^2 - t^{2q}dx_2^2 - t^{2r}dx_3^2 \,, \tag{74}$$

where p, q and r are parameters. The Kasner cosmology corresponds to vacuum (empty) cosmological model where the numbers p, q and r satisfy the constraints

$$p + q + r = 1 \,, \;\; p^2 + q^2 + r^2 = 1 \,. \tag{75}$$

The above equations are determined via the Einstein equations. The space becomes anisotropic if at least two of the three p, q and r are different.

Next, we describe an extension of the 4-dimensional Kasner cosmology to the $(4+D)$-dimensions and the metric is given by [3, 12]

$$ds^2 = dt^2 - a(t)^2 d\vec{x}^2 - c(t)^2 \left(dz_1^2 + \cdots + dz_D^2\right) . \tag{76}$$

Here $a(t)$ and $c(t)$ represent the scale factor of the 3-space and that of the extra D-space, respectively. This metric corresponds to $n \equiv 1$, $b \equiv 0$ and y-independence of a and c in Eq.(2). Moreover, there are no contributions of the bulk and the brane due to the emptiness. We can rewrite the Einstein equations by performing the appropriate linear combinations of Eqs.(7), (8) and (10)

$$\begin{aligned}
&3\frac{\ddot{a}}{a} + D\frac{\ddot{c}}{c} = 0 \,, \\
&\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + D\frac{\dot{a}}{a}\frac{\dot{c}}{c} = 0 \,, \\
&\frac{\ddot{c}}{c} + (D-1)\left(\frac{\dot{c}}{c}\right)^2 + 3\frac{\dot{a}}{a}\frac{\dot{c}}{c} = 0 \,.
\end{aligned} \tag{77}$$

We take the power-law form (so-called Kasner solution) as

$$\begin{aligned}
a(t) &= a_0 \left(\frac{t}{t_0}\right)^p \,, \\
c(t) &= c_0 \left(\frac{t}{t_0}\right)^q \,,
\end{aligned} \tag{78}$$

where a_0, c_0 and t_0 are constants and the scale factors are normalized to be zero at $t = 0$. The exponents p and q are subject to the constraints as

$$\begin{aligned}
3p + Dq &= 1 \,, \\
3p^2 + Dq^2 &= 1 \,.
\end{aligned} \tag{79}$$

The above equations can be simply checked by the substitution of Eq.(78) into Eq.(77). Solving it, we have

$$\begin{aligned}
p &= \frac{3 \pm \sqrt{3D(D+2)}}{3(D+3)} \,, \\
q &= \frac{D \mp \sqrt{3D(D+2)}}{D(D+3)} \,.
\end{aligned} \tag{80}$$

Taking the upper sign in Eq.(80), these solutions describe the case where the scale factor $a(t)$ of 3-dimensional space expand while $c(t)$ of the extra D-dimensional space shrink.

In the case of zero bulk cosmological constant, the form of the metric functions with y-dependence in Eq.(66) resembles the Kasner solutions with t-dependence in Eq.(78). Moreover, the Kasner solutions with radion potential in the framework of large extra dimensions are discussed in Ref. [3].

References

[1] P. Hořava and E. Witten, Nucl.Phys.**B460**(1996)506 [hep-th/9510209] ; Nucl.Phys.**B475**(1996)94 [hep-th/9603142]

[2] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys.Lett.**B429**(1998)263 [hep-ph/9803315]
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys.Lett.**B436**(1998)257 [hep-ph/9804398]

[3] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and J. March-Russell, Nucl.Phys.**B567**(2000)189 [hep-ph/9903224]

[4] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [hep-ph/9905221].

[5] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999) [hep-th/9906064].

[6] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and N. Kaloper, Phys.Rev.Lett. 84(2000)586 [hep-th/9907209]
V.A. Rubakov, hep-ph/0104152

[7] N. Kaloper and A. Linde, Phys.Rev.**D59**(1999)101303 [hep-th/9811141]
N. Kaloper, Phys.Rev.**D60**(1999)123506 [hep-th/9905210]
P. Binétruy, C. Deffayet and D. Langlois, Nucl.Phys.**B565**(2000)269 [hep-th/9905012]
C. Cséki, M. Graesser, L. Randall and J. Terning, Phys.Rev.**D62**(2000)045015 [hep-ph/9911406]
P. Kanti, I.I. Kogan, K.A. Olive and M. Pospelov, Phys.Rev.**D61**(2000)106004 [hep-ph/9912266] ;
Phys.Lett.**B468**(1999)31 [hep-ph/9909481]
A. Lukas, B.A. Ovrut and D. Waldram, Phys.Rev.**D61**(2000)023506 [hep-th/9902071]
H.B. Kim and H.D. Kim, Phys.Rev.**D61**(2000)064003 [hep-th/9909053]
H. Stoica, S.-H.H. Tye and I. Wasserman, Phys.Lett.**B482**(2000)205 [hep-th/0004126]
J.M. Cline, C. Grojean and G. Servant, Phys.Rev.Lett.**83**(1999)4245 [hep-ph/9906523]
H.B. Kim, hep-ph/0102182
J. March-Russell, hep-ph/0012151
D. Langlois, Phys.Rev.Lett.**86**(2001)2212 [gr-qc/0102007]
J.M. Cline, Phys.Rev. **D61**(2000)023513
L. Anchordoqui, C. Nunez and K. Olsen, **JHEP** 0010(2000)050

[8] P. Kanti, K.A. Olive and M. Pospelov, Phys.Rev.**D62**(2000)126004 [hep-ph/0005146]
J. Lesgourgues, S. Pastor, M. Peloso and L. Sorbo, Phys.Lett.**489**(2000)411 [hep-ph/0004086]

[9] M. Ito, in preparation

[10] W.D. Goldberger and M.B. Wise, Phys.Rev.**D60**(1999)107505 [hep-ph/9907218]
W.D. Goldberger and M.B. Wise, Phys.Rev.Lett.**83**(1999)4922 [hep-ph/9907447]
W.D. Goldberger and I.Z. Rothstein, Phys.Lett.**B491**(2000)339 [hep-th/0007065]

[11] R. Hofmann, P. Kanti and M. Pospelov, hep-ph/0012213
H. Collins and B. Holdom, Phys.Rev.**D63**(2001)084020 [hep-th/0009127]

[12] E. Kasner, Am. J. Math 43(1921)217
R.D.B. Collins, A.D. Matrin and E.J. Squires, "Particle Physics and Cosmology"(Wiley-Interscience, 1989)